# Optimal Control by Heat Flow in Continuous Casting Steel

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**Abstract**

An optimal control problem of the cooling in the continuous casting steel is considered. The quality of a steel ingot essentially depends from the temperature gradients in the hardening process. Minimizing the temperature gradients is the purpose of the control. The objective functional is introduced in a hard phase of the cylindrical ingot. The control being at the bound of the ingot. There is a restriction for temperature at the end of the ingot. It was realized by a penalty functional. The objective functional was minimized directly by the extreme method with regulating direction of descent.

## 1 Statement of Problem

The optimal control problems by the heat process in the continuous casting steel is actual for the metallurgy. The quality of a steel ingot is essentially defined by the thermal stresses. The breaks are formed under the high thermal stresses. The thermal stresses depend directly from the temperature gradients in a harden part of the ingot. To obtain the high quality ingots it is necessary to define the cooling regime, which gives the minimum to the temperature gradients in a hard phase of the ingot.

Consider the steadied vertical flood of a steel cylindrical ingot with the radius $R = 0.1\,m$ down to $Z = 30\,m$ (Fig.1,a). The ingot goes past two zones. Zone 1 (the crystallizer) forms a solid surface of the ingot, here the vertical coordinate $z \in (0, z_c = 0.9\,m)$. Zone 2 cools off the ingot, here $z \in (z_c, Z)$. The ingot has three states: liquid phase (white color on Fig.1,a); two-phase zone (dashed); hard phase (white and black points).

The distribution of the temperature in the ingot is described by the nonlinear elliptic equation (Sobolev, 1984):

$$(1) \qquad C\rho V \frac{\partial T}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right) = 0, \text{ over } \Sigma = \{r, z : 0 < r < R,\, 0 < z < Z\},$$

where $T(r,z)$ is the temperature, $V = 0.013\,m/s$ is the velocity of casting, $\rho(T)$, $C(T)$, $\lambda(T)$ are the mass density, the effective specific heat and the heat conductivity (Nedopekin, 1994):

$$\begin{cases} \rho = \rho_l & C = C_l; & \lambda = \lambda_l & : T \geq T_L, \\ \rho = (\rho_l + \rho_s)/2; & C = C_l + (C_s - C_l)\xi - W\dfrac{\partial T}{\partial \xi}; & \lambda = (\lambda_l + \lambda_s)/2 & : T_S < T < T_L, \\ \rho = \rho_s; & C = C_s; & \lambda = \lambda_s & : T \leq T_S, \end{cases}$$

where $\xi = 1 - \left(\dfrac{T_0 - T}{T_0 - T_L}\right)^{-0.17}$ is the portion of a hard phase in the given point of the ingot, $T_0 = 1818\,K$ is the flood temperature at $z = 0$, $T_S = 1768\,K$ and $T_L = 1798\,K$ are the initial and the final temperature of a melting still, $W = 212\,J/kg$ is the latent heat of the phase change. The indexes "$l$" and "$s$" correspond liquid and a hard state of a steel, respectively.

The boundary conditions have the form:

$$\left.\frac{\partial T}{\partial r}\right|_{\substack{r=0\\0<z<Z}} = 0, \qquad T|_{\substack{0 \le r \le R\\ z=0}} = T_0, \qquad \lambda_s \left.\frac{\partial T}{\partial r}\right|_{\substack{r=R\\0<z<z_c}} = -\gamma(T - T_c),$$

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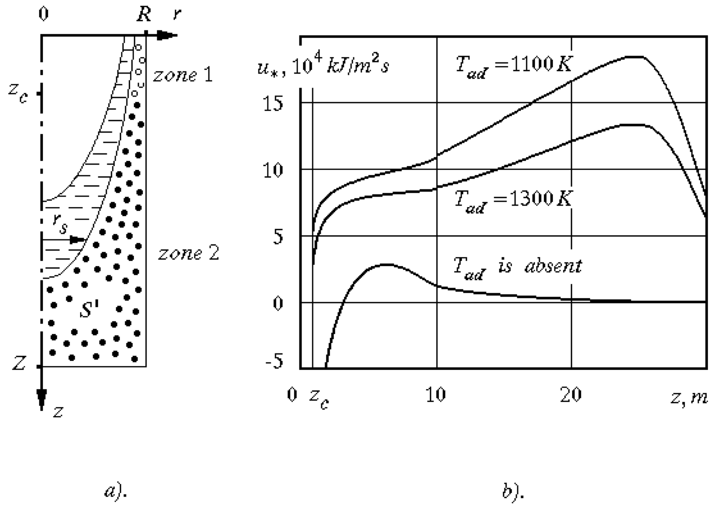


Figure 1: Continuous casting: a) — the schema of the ingot; b) — the optimal heat flow from zone 2 at $R$ with the diverse temperature restrictions at $Z$

(2)
$$\lambda_s \left.\frac{\partial T}{\partial r}\right|_S = u, \qquad S = \{r, z : r = R,\, z_c < z < Z\},$$

where $T_c$ is the temperature of the crystallizer, $\gamma$ is the heat transfer coefficient of the crystallizer, $u(z)$ is the heat outflow, which controls the cooling of the ingot.

The optimal control problem is concluded in minimizing the square of the radial components of the temperature gradients in the hard phase $S'$ of zone 2 (black points on Fig.1,a), i.e. it is necessary

(3)
$$J_0 = \int_{S'} \left(\frac{\partial T}{\partial r}\right)^2 dS \to \min, \qquad dS = r\,dr\,dz.$$

According to the technical requirements the maximum output temperature (admissible temperature) at the end of the ingot is $T_{ad} = 1100\,K$. So, we have the restriction

(4)
$$T| \le T_{ad}.$$

The restriction (4) was realized by the external penalty

(5)
$$J_p = \int_0^R I_p\,dr, \qquad I_p = \begin{cases} (T - T_{ad})^2\big|_Z, & T > T_{ad}; \\ 0, & T \le T_{ad}. \end{cases}$$

So, we have to find the heat flow $u(z)$ in condition (2), which minimizes the functional

(6)
$$J(u) = \int_0^R \int_0^Z \left(\frac{\partial T}{\partial r}\right)^2 \theta(r - r_s)\theta(z - z_c) r\,dr\,dz + \zeta \int_0^R I_p\,dr \to \min,$$

where $\theta$ is the Heaviside function, $\zeta = 10^4$ is the weight coefficient for the penalty functional.

## 2 Solving the Problem

The optimization problem (1), (2), (6) was solved by the direct extreme method (Tolstykh, 1986,1991,1996) from the gradient $\nabla J$. The gradient of the functional (6) is

$$\nabla J = -f \text{ on } S,$$

where $f$ satisfies to adjoint problem over $\{r, z : 0 < r < R,\ z_c < z < Z\}$:

$$C\rho V \frac{\partial f}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial f}{\partial r}\right) - V \frac{\partial C\rho}{\partial T}\frac{\partial T}{\partial z} f + 2\left[\left(r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\right)\theta(r - r_s) + r\frac{\partial T}{\partial r}\delta(r - r_s)\right]\theta(z - z_c) = 0. \tag{7}$$

The boundary conditions:

$$\left.\frac{\partial f}{\partial r}\right|_{\substack{r=0 \\ z_c < z < Z}} = 0, \qquad \lambda_s \left.\frac{\partial f}{\partial r}\right|_{\substack{r=R \\ z_c < z < Z}} = -2\frac{\partial T}{\partial r},$$

$$f\Big|_{\substack{0 < r < R \\ z = Z}} = -\frac{2\zeta}{C_s \rho_s V}(T - T_{ad})\theta(T - T_{ad}). \tag{8}$$

The direct extreme algorithm for searching the optimal control $u_*(z)$ has the form (Tolstykh, 1991, 1995):

$$u^{k+1} = u^k - b^k \alpha^k \nabla J^k \quad \text{on } S, \tag{9}$$

where $k$ is the iteration. The number $b^k > 0$ defines the depth of descent in the direction $\alpha^k \nabla J^k$. The function $\alpha^k(z) > 0$ on $S$. If $\alpha^k = 1$, then the algorithm (9) is the gradient method. The function $\alpha^k(z)$ regulates the direction of descent and may provide the quick uniform convergence to an optimal control (Tolstykh, 1986). The method (9) gives the opportunity to adapt the minimizer direction to the concrete objective functional. The adaptation is made by the function $\alpha$.

The parameters $b^k$ and $\alpha^k$ was chosen by new necessary and sufficient conditions for optimality (Tolstykh, 1995). In particular for $b^k$ it was used the method

$$\begin{cases} \text{if } J^k < J^{k-1}, & \text{then } b^k = b_1 b^{k-1}, b_1 \ge 1,\ k \ge 1; \\ \text{if } J^k \ge J^{k-1}, & \text{then repeat previous iteration until } J^k < J^{k-1} \\ & \text{for } b^{k-1} = b_2 b^{k-2},\ b_2 \in (0,1),\ k \ge 2. \end{cases}$$

There was adopted $b_1 = 1.2$, $b_2 = 0.3$. The function $\alpha^k$ was adapted on the first iteration from the following condition

$$\alpha^k(z) = \frac{0.2u^0(z)}{|\nabla J^0(z)|}, \qquad \text{where the initial guess is} \qquad u^0(z) = 200\frac{kJ}{m^2 s}.$$

The sense of this adaptation is concluded in following. For the constant initial guess, which is far from the optimal value, we have to have the constant next approximation $u^1$. Under $b^0 = 1$ the function $\alpha$ gives constant change 20% on $S$ of the control $u^0$.

On Fig.1,b have been taken the results of the optimal controlling. The algorithm (9) has converged in practice for 20 iterations. For the penalty temperature $T_{ad} = 1100\,K$ the functional $J$ has decreased in 1.8 time, and the criterion of the thermal stresses (3) — in 1.9 time. If we raise the penalty temperature up to $T_{ad} = 1300\,K$, we will have the less heat outflow and $J_0^0/J_0^{20} = 2.7$, i.e. we will decrease the thermal stresses. At the end ($z > 25\,m$) outside of the ingot the temperature does not violate the restriction (4), therefore the heat flow can be decreased for minimizing the thermal stresses here. These curves have the physical sense.

If the restriction (4) is absent we will have $J_0^0/J_0^{10} = 30$ and the new optimal control $u_*$ in principle. Here a hard phase in zone 2 is present near the crystallizer only under $S' \approx 0$. The other part of zone 2 has a two-phase state, where $\partial T/\partial r = 0$ and $T = T_S$.

So, we found the optimal regimes for cooling the continuous ingot. The temperature restrictions essentialy influence on the view of the optimal control $u_*(z)$ and on the value of the thermal stresses in the ingot.

# References


[1] Sobolev V.V. and Trefilov P.M.(1984). *The process of heat and mass transfers and harding in the ingot (Russian),* Krasnoiarsk Univ.

[2] Nedopekin F.V. (1994). *Mathematical modelling the hydrodynamic and heat and mass transfers in the ingot (Russian),*Udmurtsk Univ.

[3] Tolstykh V.K. (1986). *Applying the gradient method to problems of optimization of the distributed parameter systems (Russian),* J˙ of Calcul˙ Mathem˙and Mathem˙Physics, **26**, 1, 137-140. Nauka, Moscow.

[4] Tolstykh V.K. (1991). *The gradient method of optimal control by distributed systems (in Russian),* Differential Equations, **27**, 2, 303-312. Navuka i Technika. Minsk.

[5] Tolstykh V.K. (1995). *Direct extreme approach in control theory for distributed systems,* Abs. of Sump. on Operation Research, Passau - Germany, p.114.